Standardization of a Farad

*Electrolytic capacitors with capacitance values as great as
one farad are available, and bridges are also available with which to measure them.
However, in using a bridge to measure large capacitances, small mutual inductances
between internal and external bridge leads may produce appreciable errors. Standards
are described that can be used to determine bridge errors, correction terms
are presented, and possible measurement errors are discussed.*

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Electrolytic capacitors as large as one farad are available commercially, and bridges are available to measure them [1]. Moreover, a standard of this value is also available [2]. While the stated ±10% accuracy of the electrolytic capacitors is not too great, the standard has a substantially better accuracy of 1%. Although 0.5% accuracy could be considered adequate for a bridge used to calibrate a 1% standard capacitor, if the adjustment tolerance and stability of the standard were well within the remainng 0.5%, it would be desirable to reduce the bridge error to 0.1%. Unfortunately, NBS is not able to calibrate capacitors of this value, so their facilities could not be employed for a calibration to determine bridge error.

A capacitance of one farad exhibits a low impedance—even to low frequencies. At the standard frequency of 120 Hz that is used for checking polarized electrolytic capacitors, the impedance of a 1-farad capacitor is approximately 1.3 milliohms. A four-terminal bridge is necessary to avoid not only dissipation-factor errors, but also capacitance errors due to lead inductance. A commercial, four-terminal bridge of this type, that tolerates a resistance of 100 milliohms in any or all leads (which will cause less than 1% measurement error) is shown below, together with a 1-μF capacitance standard.

The precision bridge used for calibration, as shown in Fig. 1, is a high-resolution version of the same circuit, with external power supply and detector. This bridge uses an injecting transformer (sim-
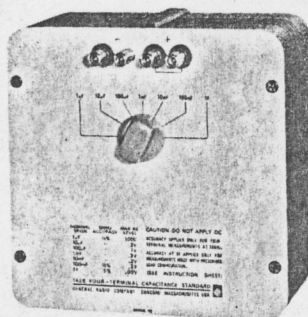
● The Four-Terminal Capacitance Standard, Type 1426 (left), consists of a 1-μF polystyrene capacitor and a transformer that multiplies the effective capacitance. It provides capacitance values from 1 μF to 1 F in seven switch-selected decade values. At a frequency of 120 Hz and a temperature of +23°C, its rated accuracy is ±0.25% for all values except 100 mF and 1 F, for which it is, respectively, ±0.5% and ±1%. ● The Type 1617-A Capacitance Bridge (right) is intended specifically for measuring the capacitance, dissipation factor, and leakage current of electrolytic capacitors. Completely self-contained, it includes a 120-Hz generator, a null detector, a DC polarizing-voltage supply, and metering for bias voltage and leakage current. The bridge includes an Orthonull® balance finder, which eliminates "sliding balance". Its rated accuracy, at a frequency of 120 Hz, is ±1% ± 1 pF—except that for capacitance values from 110 mF to 1.1 F, the accuracy becomes ±2%.

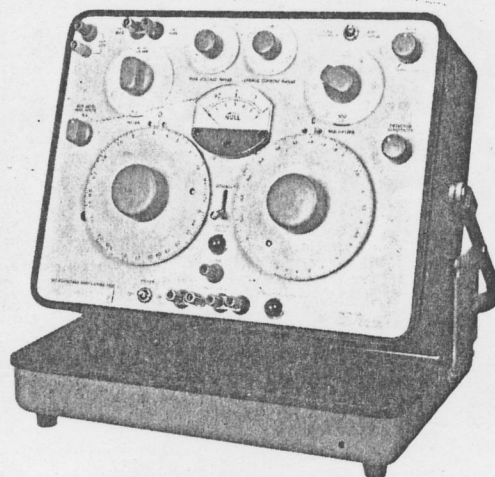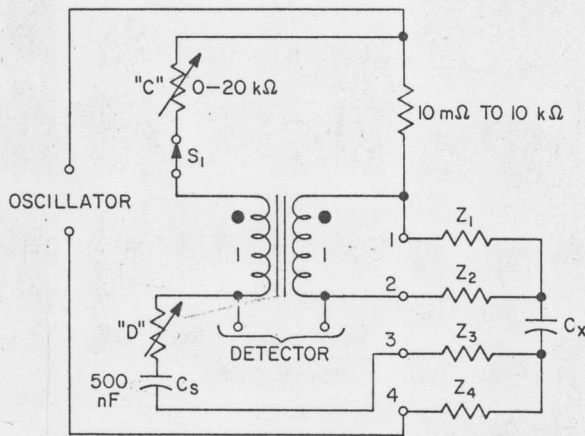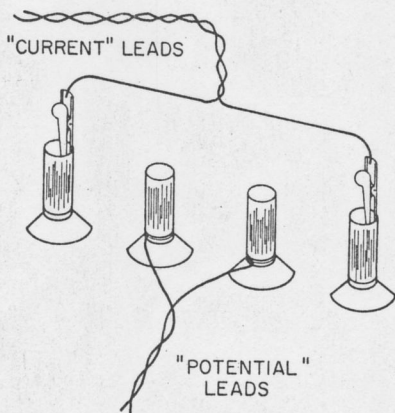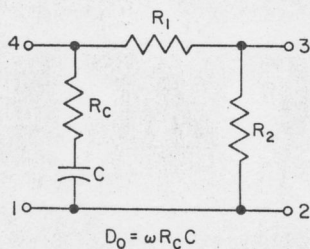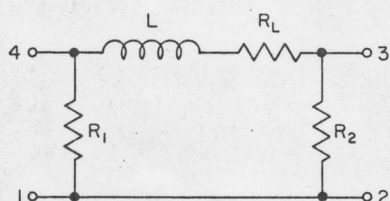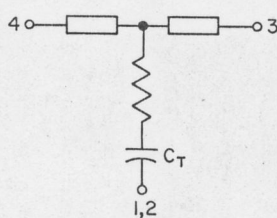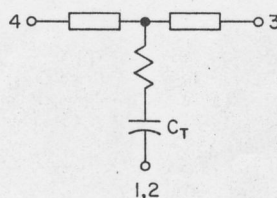ilar to those used by Foord, Langlands, and Binnie [3]) to avoid the error due to voltage drop in what might be called the "yoke"—by analogy to a Kelvin Bridge. Any one-farad standard capacitor must also be a four-terminal device, to avoid connection errors.

The Problem

● *Initial Calibrations.* The first calibrations of the 1-farad standard were based on two techniques: using the bridge as a standard; and using a decade-scaling procedure that compared accuracies at full scale and at one-tenth full scale. In the first technique, the bridge circuit was set up using separate standards and a decade resistor, with a variety of known component values for the bridge arms. The measured value of the 1-farad standard capacitor remained well within 1%. The actual bridge was assembled in a permanent form and its components set to within 0.01%, and again the 1-farad standard was measured within the 1% range. It can be argued that the bridge itself is a valid standard if its component values are known, because they have a known relationship to the unknown.

The scaling procedure was based on the accuracy of the ratio between the bridge full-scale setting and the setting at 1/10 of full scale, which was checked by measuring known standards of lower value. Then, measuring standards of decade values on two bridge ranges, the accuracy both of the standards and of the bridge ranges could be checked. Starting with a known $1\text{-}\mu\text{F}$ standard, this procedure also indicated that the 1-farad measurement was within 1%.

The measurements described below indicate that these initial calibrations were indeed within 1%. It now appears that this precision bridge read high by about 0.4%, due to mutual inductance.

● *The Mutual-Inductance Error.* While a four-terminal measurement can avoid errors due to self-inductance of leads, mutual inductance between the "current" and

"potential" leads results in an induced voltage on the potential leads, causing the effective value to be changed [1]. The value, $C_m = C_x/(1 - \omega^2 M C_x)$ will be measured. If one-foot-long leads are used, and are spread to form a circle, this mutual inductance is approximately 250 nH (2.5×10^{-7} henry), which would cause a 15% error at a frequency of 120 Hz. This error can be reduced greatly by twisting the current (or potential) leads together, and if this is done, the error due to lead geometry is well within 1%. It should be noted that the mutual inductance can be positive or negative, depending on the lead geometry.
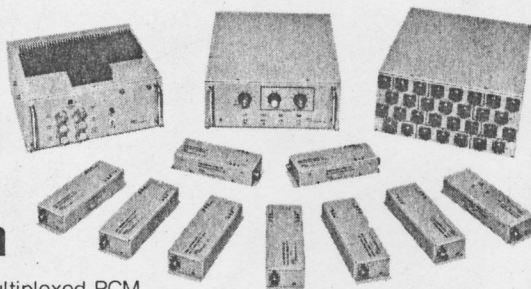
Ideally, the solution to this problem would be to use a completely coaxial system. Practically, this makes little sense, because sooner or later the connection problem has to be faced. The eventual objective of all these calibrations is to measure actual electrolytic capacitors, which are rarely encountered with coaxial, four-terminal connections.

If tightly twisted leads are used, mutual inductance between the leads is avoided, but it still remains in four places: inside the bridge; at the bridge terminals (binding posts); at the unknown terminals; and inside the unknown. The mutual inductance inside the bridge should be known and its effects corrected, or else it should be reduced to a negligible value by suitable arrangement of the internal leads. The mutual inductance at the bridge terminals, using a repeatable lead configuration, should also be determined and/or compensated. The repeatable configuration that was used is shown in Fig. 2a, where the "potential" loop is small, and the "current" loop is at right angles to it for minimum coupling.

The mutual inductance at the binding-post terminals of the 1-farad standard capacitor can be accounted for by "defining" the calibrated capacitance as that measured with the repeatable connection of Fig. 2a. For a two-terminal capacitor, the correct value could be defined as that
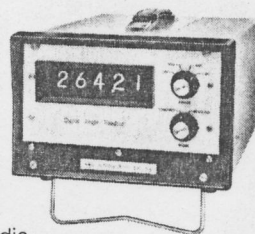
to the standard must be accounted for, or else the calculated value will not agree with the measured value.*

"Calculable" Networks as Standards

While the bridge of Fig. 1 can tolerate only small values of the impedances Z_1 and Z_2, it can tolerate quite high values for Z_3 and Z_4. Impedance Z_3 is in series with the bridge standard, which is a 500-nF capacitor that exhibits an impedance of approximately 2600 ohms at a frequency of 120 Hz. Lead-inductance errors are almost completely negligible, and lead resistance produces an error in the value of D, but not in that of C. Impedance Z_4 is in series with the source and, theoretically, may be of any value. However, it does reduce sensitivity, and, if not completely shielded, can result in errors due to small stray capacitances. This high immunity to lead errors permits the use of standards that are (passive) three-terminal networks. Any such network can be transformed into an equivalent T (or Y) network, whose mutual impedance (leg) can be used as the standard, while the remaining impedances (arms) become Z_3 and Z_4. Three such networks were used.

● *The RC Network.* The delta network of Fig. 3a can be transformed into the T network of Fig. 3b. If we let D_o be the dissipation factor of the capacitor, and D_m the measured dissipation factor of the resulting standard, the effective value of the standard is (very nearly)

$$C_T \approx \frac{(R_1 + R_2)(1 + xy)}{R_2} \quad (1)$$

where $x = D_m - D_o$, and $y = D_m + D_o$
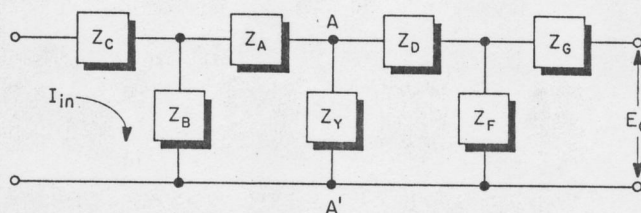
The resistors used were noninductive, and a four-terminal con-

measured with twisted leads that connect at right angles, as shown in Fig. 2b. In both cases, the mutual inductance inside the capacitor would be considered as part of the measured value of capacitance.

For the "calculable" capacitors described below, the mutual inductance at the terminals and internal

nection was made to R_2—the smaller resistor. Five versions of this standard were used, with C values of 100 μF and 1 mF, and resistances ranging from 100 mΩ to 10 kΩ.

● *The RL Network.* The delta network of Fig. 4a can be transformed to the T network of Fig. 4b. If D_m is the measured dissipation factor of the resulting capacitance standard, $C = L(1 + D_m^2)/R_1R_2$ will be the effective value of the capacitor. Three standards were used, with inductors ranging in value from 10 mH to 1 H, and resistances of 100 milliohms and 1 ohm. The inductors used were somewhat level dependent, and measurements were made at the lowest practical excitation level.

● *The Transformer-Capacitor Network.* The network of Fig. 5a can be transformed into the T of Fig. 5b. If Z_0 is the impedance measured between points A and A' (terminals open circuited), the ratio between Z_T and Z_0 is then

$$Z_T = \frac{Z_0 N_1^2 (1 - \delta_1)(1 - \delta_2)}{N_2^2} \quad (2)$$

where δ_1 and δ_2 are the deviations in the step-down-transformer ratios from the actual turns ratios —which are easily determined. Therefore,

$$C_T \approx \frac{C_0 N_2^2}{N_1^2(1 + \delta_1 + \delta_2)} \quad (3)$$

The transformers used had turns ratios of 100-to-1, so that the capacitance value was 100 μF. A more complete analysis of the transformer-capacitor network is presented in the appendix to this article.

● *Results.* Nine different measurements performed on all these networks, when corrected by the above equations, were within a 0.2% spread in value, and averaged 0.4% high. These errors were assumed to be due to mutual inductance in the bridge and its terminal connections, and in the standard networks. Similar 100-nF standards agreed with the bridge well within a tolerance of 0.1%.
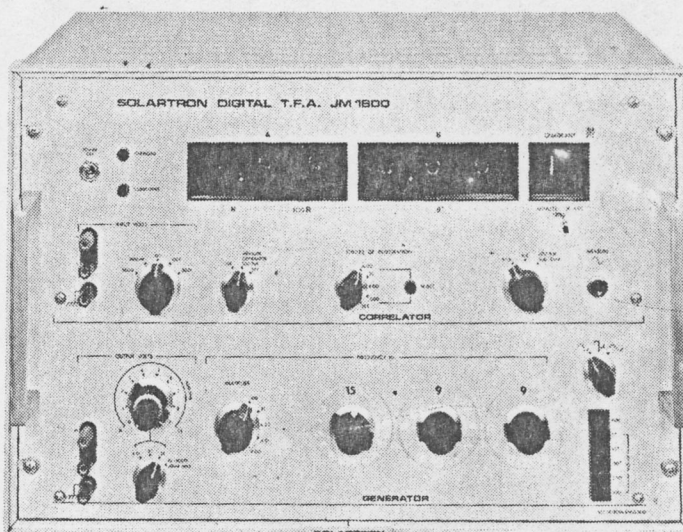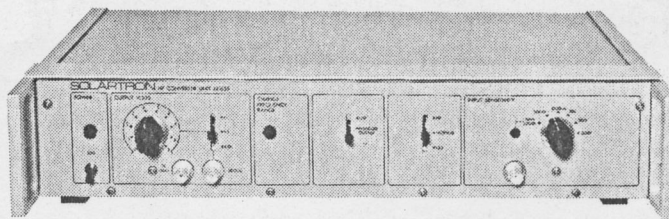
Mutual Inductance Adjustment

The actual values of very small inductance standards are defined as the difference between the measured value and the quantity measured when the inductor is shorted at the standard [4]. Lead inductance affects both measurements and cancels out, so that the same difference value is obtained, regardless of the leads that are used. A similar scheme could be employed with a capacitance bridge, but it would have to be capable of measuring lead inductance in terms of very large negative capacitance values.

An alternative is to open switch S, in Fig. 1, which will produce a null at infinite capacitance. A variable mutual inductor is then placed in the leads, as shown in Fig. 6, and adjusted for a detector null to make the initial balance. Each of the standard networks produces an infinite capacitance by shorting the standard capacitors, or by open-circuiting the inductor. When this was done, and the initial adjustment made, the measured values of the networks agreed with the calculated values within ±0.1%.

The Negative-M Standard

As a final check, two windings were placed on a ceramic toroidal form. The mutual inductance between them was determined by measuring the self-inductance of the two windings in series—first aiding, and then opposing. It can be shown that the mutual inductance is then equal to one-fourth of the difference between these two measured values. When the coils are connected as shown in Fig. 7a, they form the T network of Fig. 7b. This negative mutual inductance can be measured as a capacitance of value $C = 1/\omega^2 M$. Note that the test frequency must be determined accurately.

This standard can be set to infinite capacitance (zero M) by connecting both potential leads to the same terminal. When this was done, and the initial balance made, the measured and calculated values again agreed to within 0.1%.

Final Details

The 0.4% error due to mutual inductance was shown, by measurements made with the initial adjustment set with the "potential" leads shorted, to be almost entirely in the bridge and its terminals. The values agreed very well with measurements made with the standards set to infinity, as described above. The internal mutual inductance of the bridge was then adjusted, by placement of leads, to give the correct initial balance when the connections to the bridge were made as shown in Fig. 2. The commercial 1-farad standard capacitors are calibrated with this bridge, with the connections on both bridge and standard made as shown in Fig. 2. Thus, this value includes the mutual inductance at the terminals of the standard, but not that at the bridge.

By opening its main adjustment arm and noting the mutual inductance required for a short-circuit balance, the mutual inductance of the commercial bridge used was found to be negative. This bridge reads approximately 0.5% low for a value of one farad, with the lead configuration as shown in Fig. 2. If, instead of the lead configuration of Fig. 2, the standard lead set provided with the bridge is used, with the potential wires tightly twisted, the increase in mutual inductance provides a good cancellation of this error.

Results

A capacitance of 1 farad was measured to an error of only ±0.1% by means of networks made up of calibrated components of reasonable value. Any attempt to determine the farad more accurately must use a bridge of higher accuracy and precision, some extremely repeatable method of connection (preferably coaxial), and a careful study of all possible sources of error that might have caused the small variations among the measurements described above.

It should be emphasized that it is *very* easy to obtain errors of ±10%—and even more—using a good ±0.1% bridge, unless the connection precautions discussed are followed. Thus, particular attention must be paid to this aspect of the measurement technique.

APPENDIX: Analysis of the Transformer-Capacitor Network

Any passive, linear, three-terminal network—including a two-winding transformer with a common terminal—can be drawn as a T network. If two such T networks are connected with an impedance Z_Y between them, as shown in Fig. 8, the impedance Z_0, measured across that impedance (A to A'), will be

$$Z_0 = \frac{Z_Y(Z_A+Z_B)(Z_D+Z_F)}{(Z_A+Z_B)(Z_D+Z_F)+Z_Y(Z_A+Z_B+Z_D+Z_F)} \quad (4)$$

If a current, I_{in}, is impressed at one end of this network, and the open-circuit voltage, E_0, measured at the other, the transfer impedance, Z_T, is

$$Z_T = \frac{E_0}{I_{in}} = \frac{Z_B + Z_F + Z_Y}{(Z_A+Z_B)(Z_D+Z_F)+Z_Y(Z_A+Z_B+Z_D+Z_F)} \quad (5)$$

Therefore:

$$\frac{Z_T}{Z_0} = \frac{Z_B Z_F}{(Z_A+Z_B)(Z_D+Z_F)}$$
$$= \left[\frac{Z_B}{Z_A+Z_B}\right]\left[\frac{Z_F}{Z_D+Z_F}\right] \quad (6)$$

These last two factors are the open-circuit ratios of the two dividers formed by Z_A & Z_B, and Z_D & Z_F. If the transformers of Fig. 5 were ideal, these ratios would be simply N_1/N_2. Since the transformers are not ideal, the ratios will be slightly lower. If we call these ratios $N_1(1-\delta_1)/N_2$ and $N_1(1-\delta_2)/N_2$, then

$$\frac{Z_T}{Z_0} = \left[\frac{N_1(1-\delta_1)}{N_2}\right]\left[\frac{N_1(1-\delta_2)}{N_2}\right] \quad (7)$$

Because these deviations are small, from equation (7), Z_T can be approximated very closely as

$$Z_T \approx Z_0\left(\frac{N_1^2}{N_2^2}\right)(1-\delta_1-\delta_2) \quad (8)$$

If Z_0 is a pure capacitance and the dividers have no phase error, then

$$C_T \approx C_0\left(\frac{N_2^2}{N_1^2}\right)(1+\delta_1+\delta_2) \quad (9)$$

Actually, of course, Z_0 is not a pure capacitance, because of the shunting impedance of the dividers and the loss in the capacitor used; also, the divider exhibits some phase error. If the effective series capacitance of Z_0 has a dissipation factor, D_0, and $N_1(1+a_1+jb_1)/N_2$ and $N_1(1+a_2+jb_2)/N_2$ are the actual divider

ratios [where a_1 and a_2 are usually negative, since $a_1+jb_1=-\delta_1$], it can be shown easily that the effective series capacitance, C_T, of the transfer impedance is

$$C_T = C_0 \left[\frac{N_\beta^2}{N_\gamma^2}\right][1-a_1-a_2+D_0(b_1+b_2)]$$

(10)

The phase errors, b_1 and b_2, are small (about 0.0003), as is D_0 (about 0.016 for the network used), so that their product is negligible, and may be ignored for 0.1% measurements.

It should be noted that, ideally, all capacitance measurements and voltage-ratio measurements should be made at the same flux level, because a transformer is not a truly linear device.

Note also that the two transformers need not have the same ratios, and that these dividers could be resistive. However, resistive dividers would have to use high-valued resistors to avoid a large value of D_0, and this would result in large lead impedances, Z_3 and Z_4 (Fig. 1).

K

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3. Foord, T. R., Langlands, R. C., and Binnie, A., "Transformer-Radio Bridge Network with Precise Lead Compensation", *Proc. IEE (Eng.)*, September, 1963 (Vol. 110, No. 9); pg 86.
4. Hersh, J. R., "Connection Errors in Inductance Measurements", *GR Experimenter*, October, 1960.

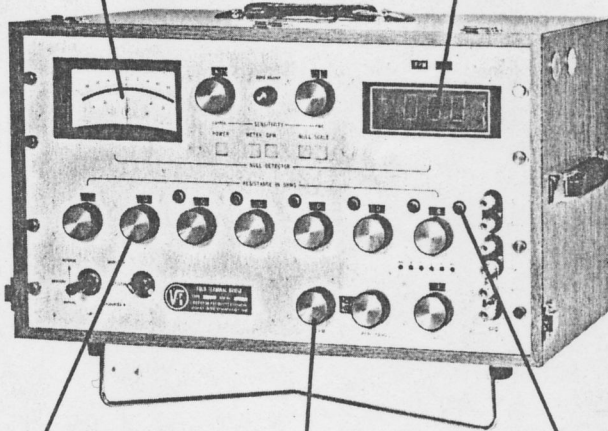Henry P. Hall *received the BA degree from Williams College, and the BSEE and MSEE degrees from MIT in 1952. He joined General Radio Company in 1949 as a student, and in 1964 became Group Leader of the engineering group responsible for the development of low-frequency impedance measuring instruments and standards. Last year he resigned as Group Leader, to become an Engineering Staff Consultant, so that he would have more time to pursue his interests and to develop new instruments. He designed many of GR's instruments and standards, including the 1650, 1608 and 1656 Impedance Bridges, the 1644 Megohm Bridge, and the 1617 Capacitance Bridge. Mr. Hall is a member of the PMA, and a Senior Member of the IEEE.*